GOVERMENT POLYTECHNIC VAISHALI

Unit -4 Moment of Inertia

Subject Code → 1615402

Subject Name → Mechanics of Structure

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Course objective : student will be able to

1) find moment of inertia of different shapes of plane areas, composite section, built-up sections, symmetrical & unsymmetrical section.
2) Understand radius of gyration & polar moment of Inertia.
3) Apply parallel axis & perpendicular axis theorem.

Moment of Inertia

What is a Moment of Inertia?

- It is a measure of an object's resistance to changes to its rotation.
- Also defined as the capacity of a cross-section to resist bending.
- It must be specified with respect to a chosen axis of rotation.
- It is usually quantified in m^4 or kgm^2

Quick Note about Centroids….

- The centroid, or center of gravity, of any object is the point within that object from which the force of gravity appears to act.
- An object will remain at rest if it is balanced on any point along a vertical line passing through its center of gravity.

and more…

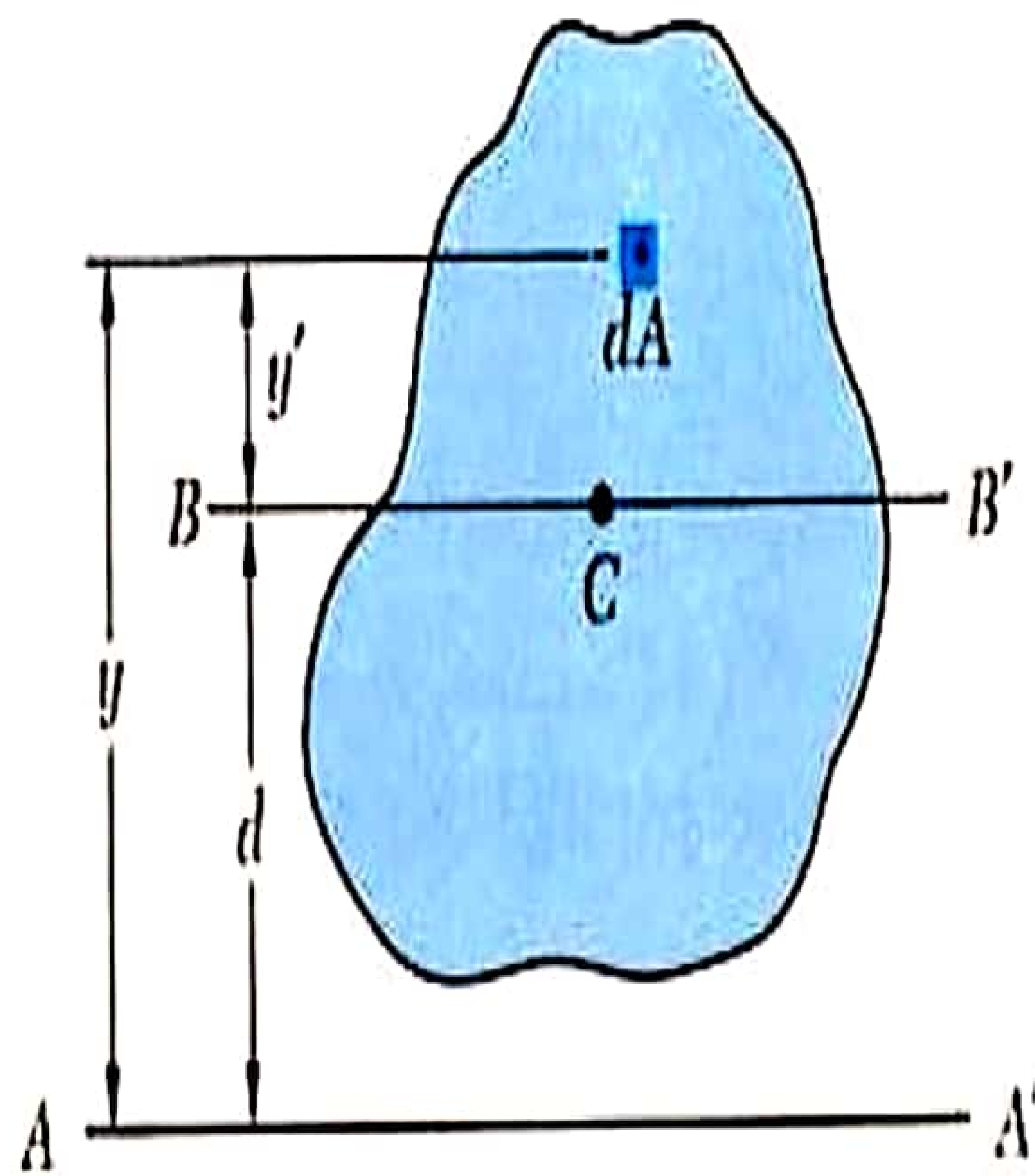
- The centroid of a 2D surface is a point that corresponds to the center of gravity of a very thin homogeneous plate of the same area and shape.
- If the area (or section or body) has one line of symmetry, the centroid will lie somewhere along the line of symmetry.

Perpendicular Axis Theorem

- The moment of inertia (MI) of a plane area about an axis normal to the plane is equal to the sum of the moments of inertia about any two mutually perpendicular axes lying in the plane and passing through the given axis.
- That means the Moment of Inertia $I_z = I_x + I_y$

Parallel Axis Theorem

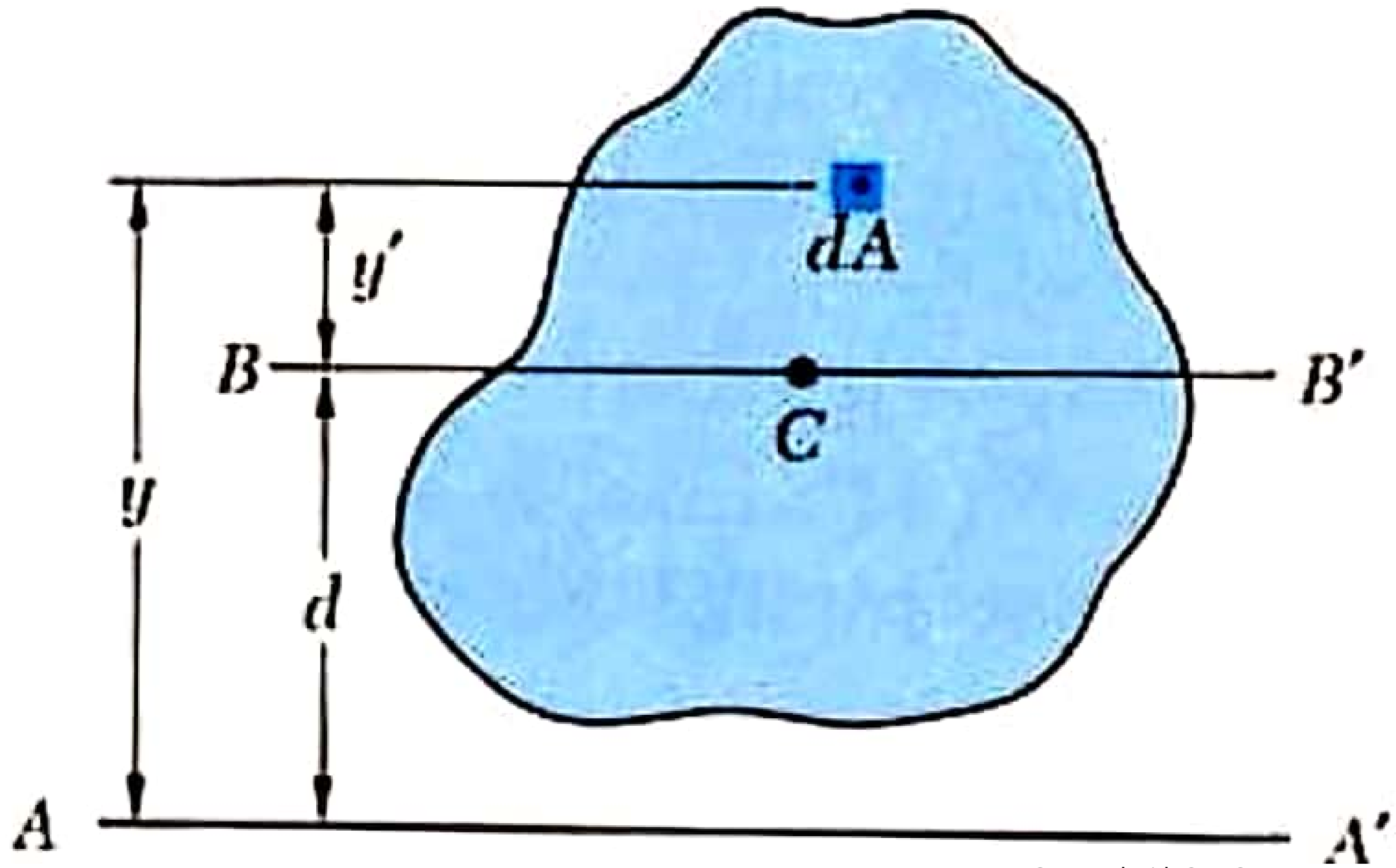
- The moment of area of an object about any axis parallel to the centroidal axis is the sum of MI about its centroidal axis and the prodcut of area with the square of distance of from the reference axis.
- Essentially, $I_{XX} = I_G + Ad^2$
- A is the cross-sectional area.
  d is the perpendicuar distance between the centroidal axis and the parallel axis.



Parallel Axis Theorem - Derivation

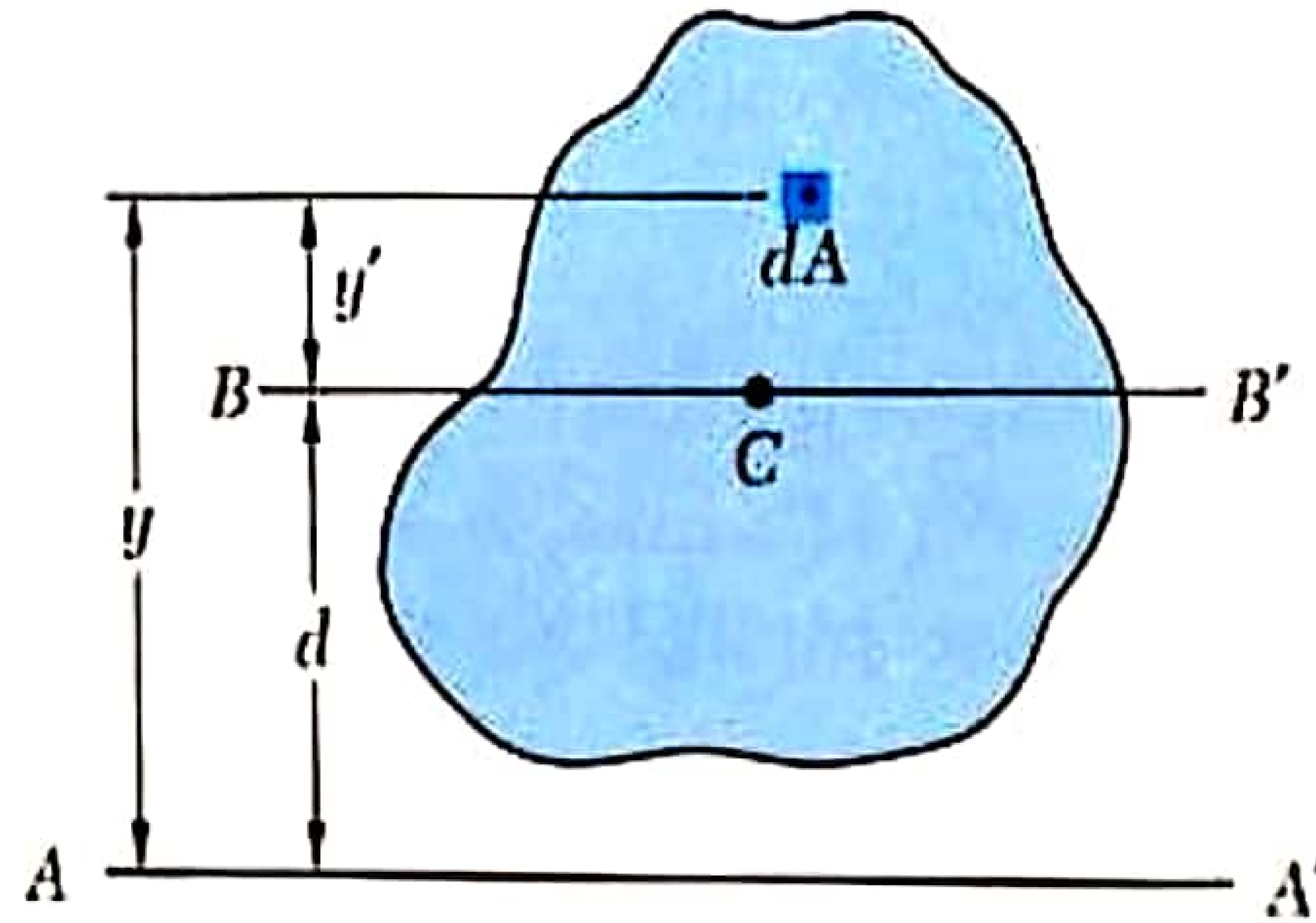
- Consider the moment of inertia I_x of an area A with respect to an axis AA'. Denote by y, the distance from an element of area dA to AA'.

$$I_x = \int y^2 dA$$



Derivation (cont'd)

- Consider an axis BB' parallel to AA' through the centroid C of the area, known as the centroidal axis. The equation of the moment inertia becomes:



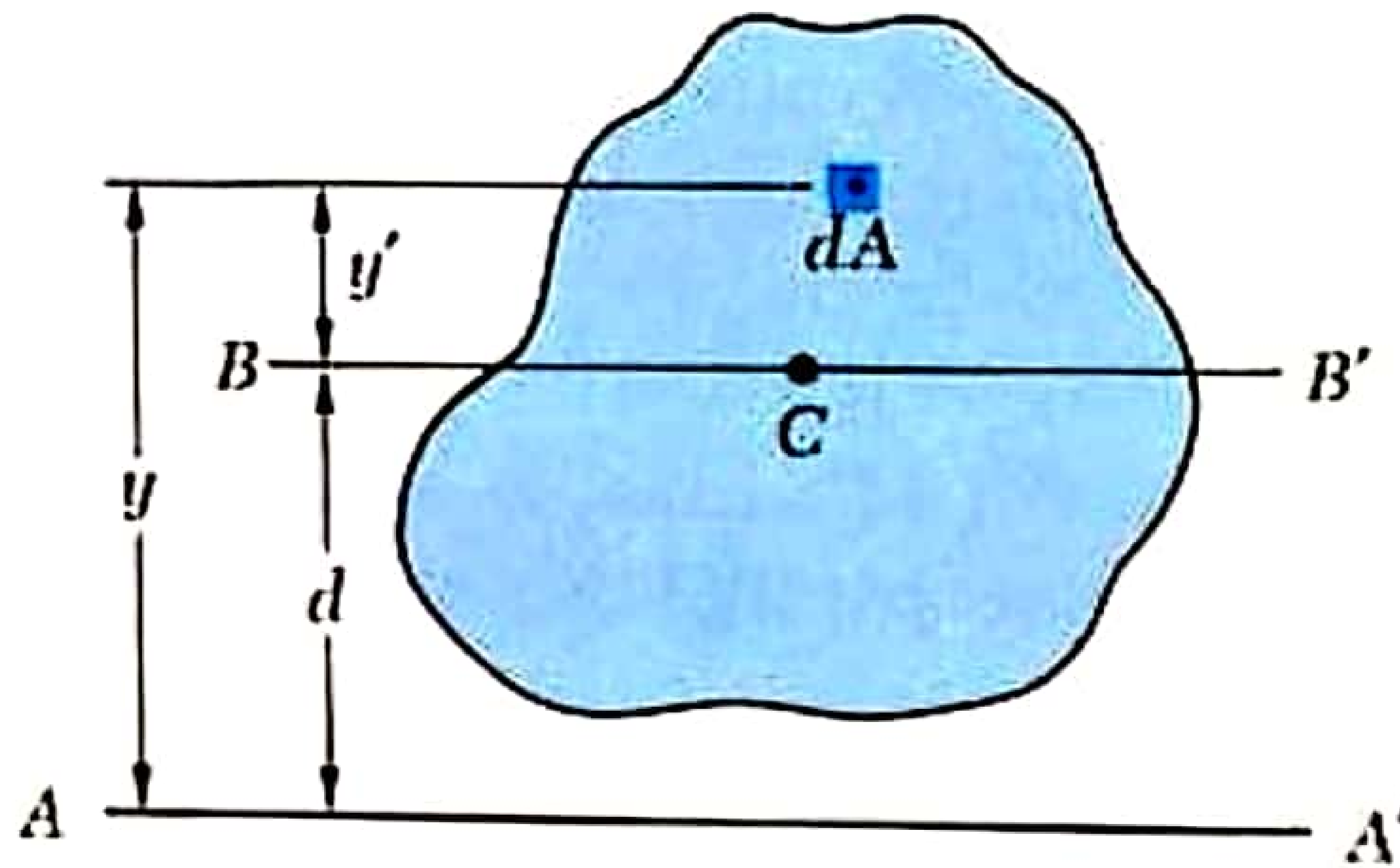
$$I_x = \int y^2 dA = \int (y' + d)^2 dA$$

$$= \int y'^2 dA + 2\int y' dA + d^2 \int dA$$

Derivation (cont'd)

The first integral is the moment of inertia about the centroid.

$$\overline{I_x} = \int y'^2 dA$$

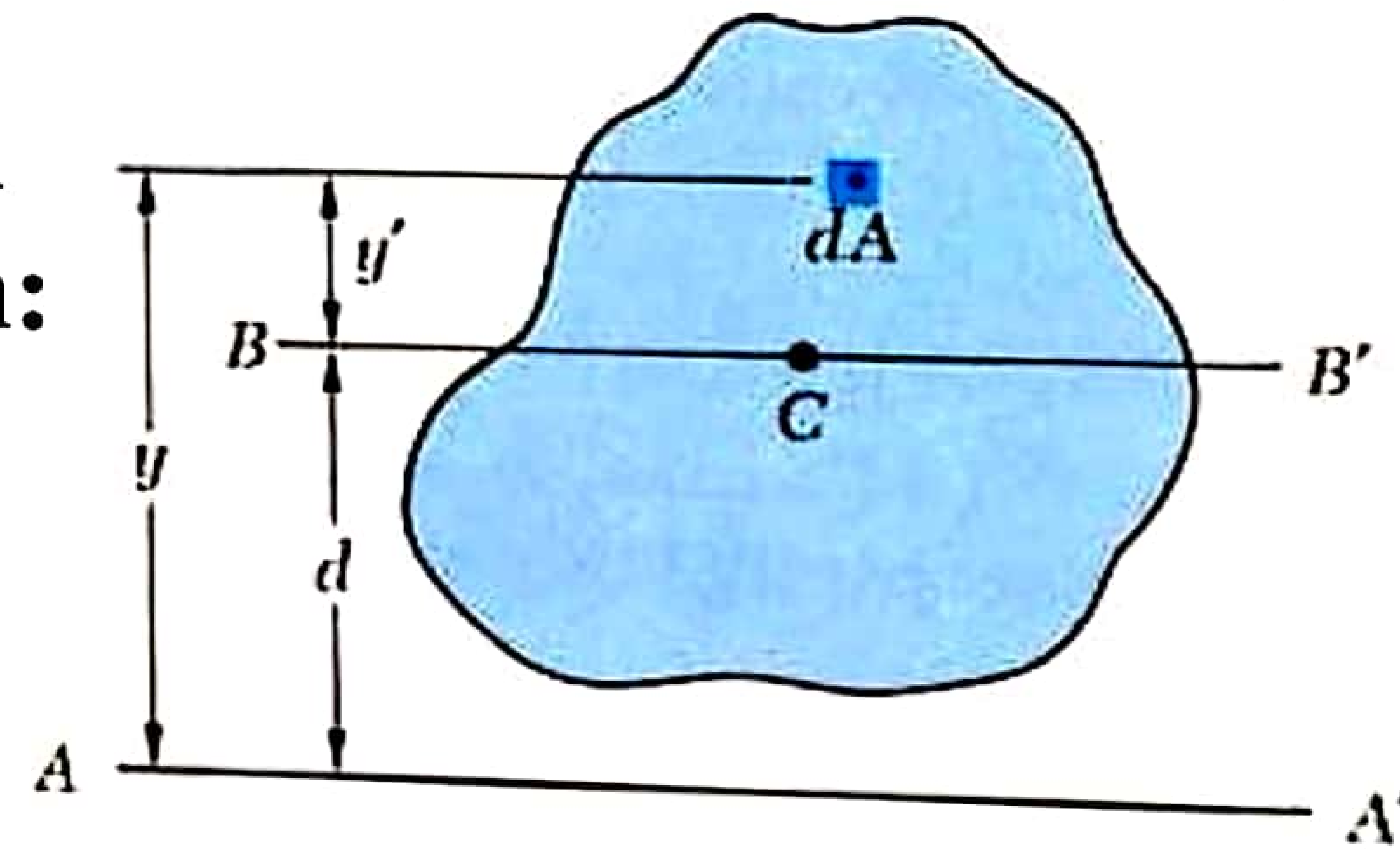


The second component is the first moment area about the centroid:

$$\overline{y'}A = \int y' dA \Rightarrow \overline{y'} =$$

Derivation (cont'd)

Modify the equation obtained with the parallel axis theorem:



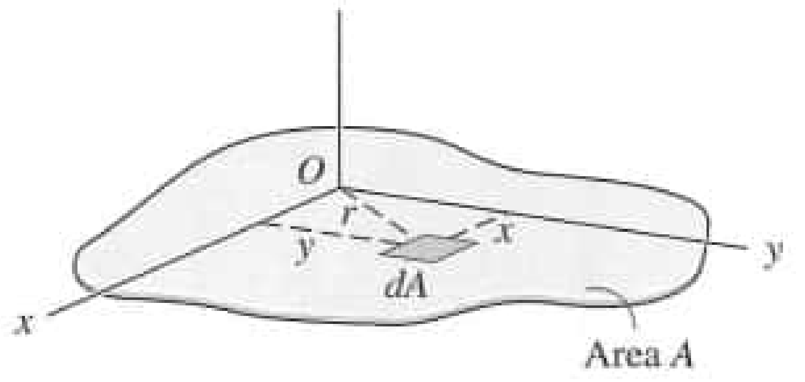
$$I_x = \int y'^2 dA + 2\int y' dA + d^2 \int dA$$

$$= \bar{I}_x + d^2 A$$

Moment of Inertia and Properties of Plane Areas

The **Moment of Inertia (I)** is a term used to describe the capacity of a cross-section to resist bending. It is always considered with respect to a reference axis such as X-X or Y-Y. It is a mathematical property of a section concerned with a surface area and how that area is distributed about the reference axis (*axis of interest*). The reference axis is usually a centroidal axis.

The moment of inertia is also known as the **Second Moment of the Area** and is expressed mathematically as:



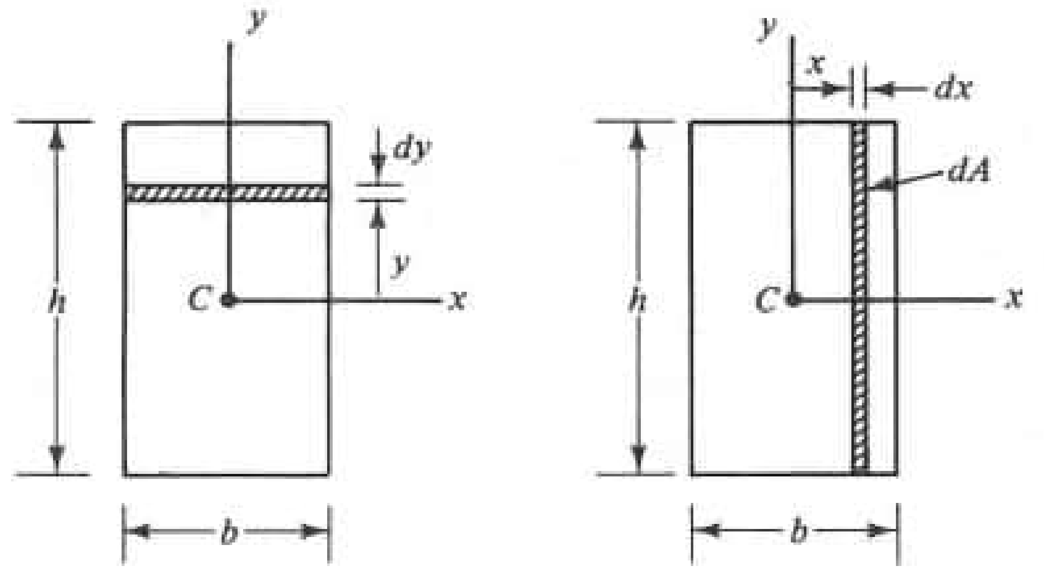
$$I_x = \int_A y^2 dA$$

$$I_y = \int_A x^2 dA$$

Where

y = distance from the x axis to area *dA*
x = distance from the y axis to area *dA*

Example



Radius of Gyration:

$$k_x = \sqrt{\frac{I_x}{A}}$$

$$k_y = \sqrt{\frac{I_y}{A}}$$

The **radius of gyration** of an area with respect to a particular axis is the square root of the quotient of the moment of inertia divided by the area. It is the distance at which the entire area must be assumed to be concentrated in order that the product of the area and the square of this distance will equal the moment of inertia of the actual area about the given axis. In other words, the radius of gyration describes the way in which the total cross-sectional area is distributed around its centroidal axis. If more area is distributed further from the axis, it will have greater resistance to buckling. The most efficient column section to resist buckling is a circular pipe, because it has its area distributed as far away as possible from the centroid.
Rearranging we have:

$$I_x = k_x^2 A$$
$$I_y = k_y^2 A$$

The radius of gyration is the distance *k* away from the axis that all the area can be concentrated to result in the same moment of inertia.

Polar Moment of Inertia:

$$I_p = \int_A \rho^2 dA$$

$$I_p = \int_A (x^2 + y^2) dA$$

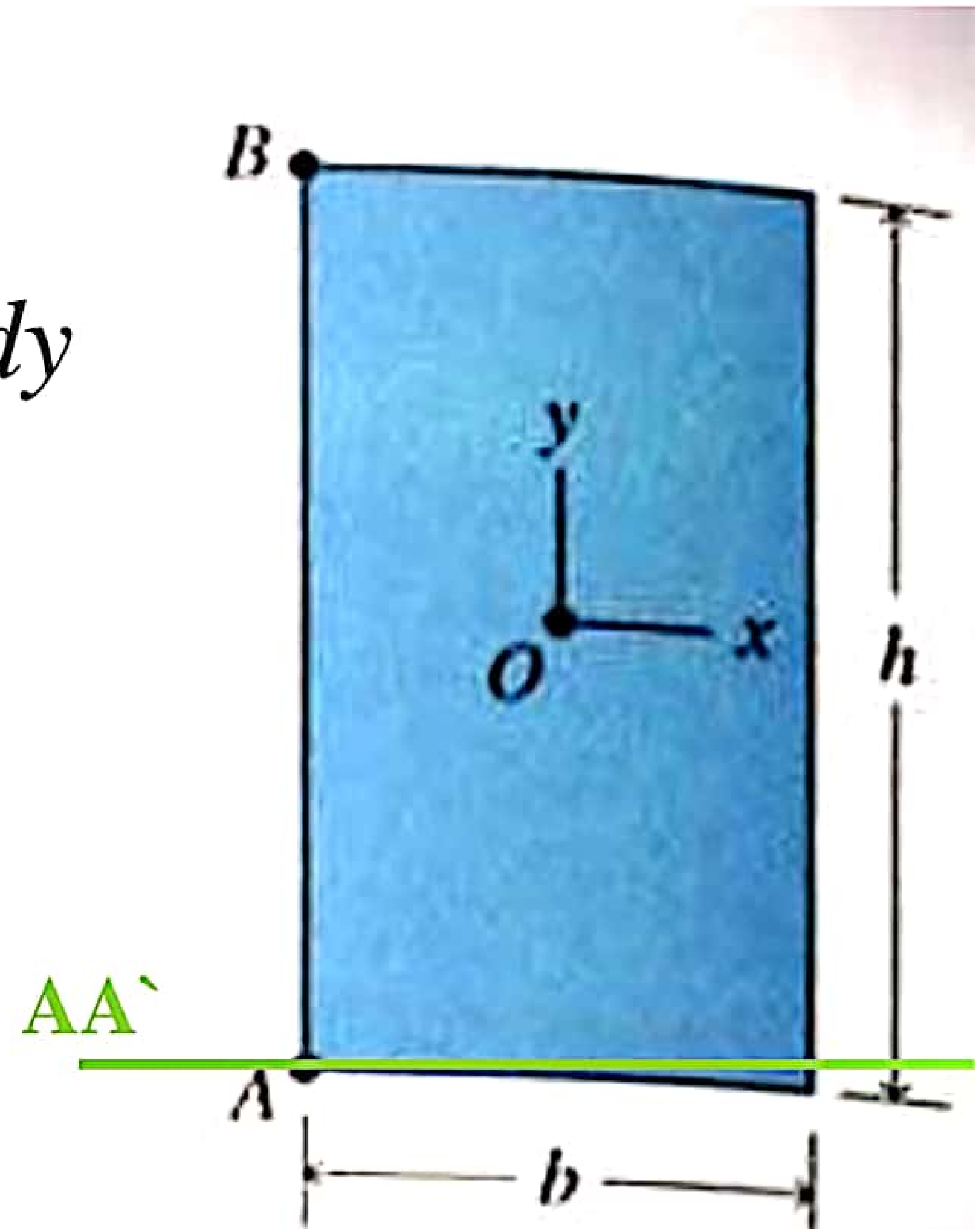
$$I_p = \int_A x^2 dA + \int_A y^2 dA$$

$$I_p = I_x + I_y$$

Example

$$I_x = \int_{Area} y^2 dA = \int_0^h \int_0^b y^2 dx dy$$

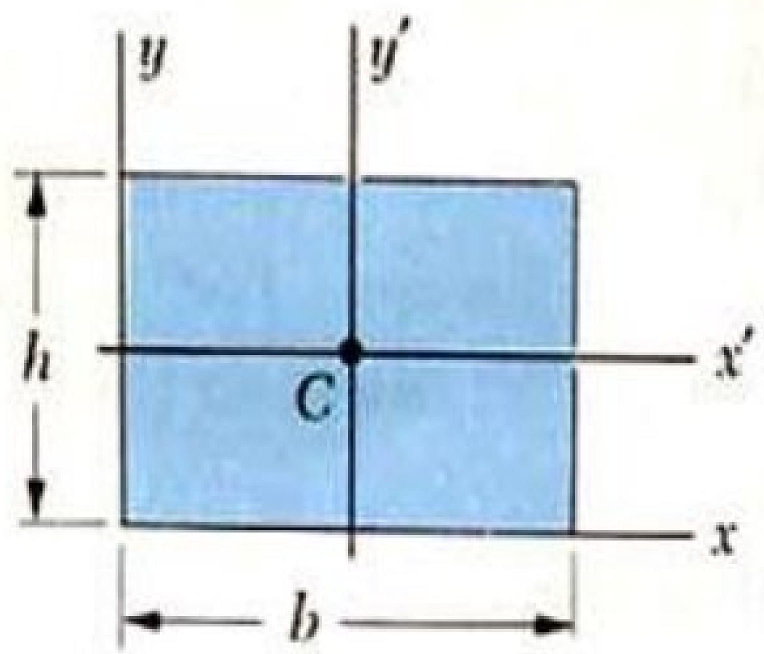
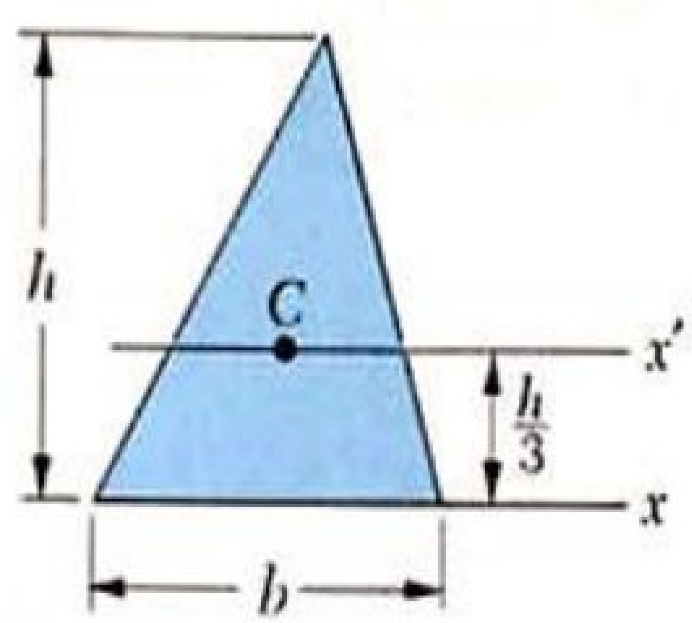
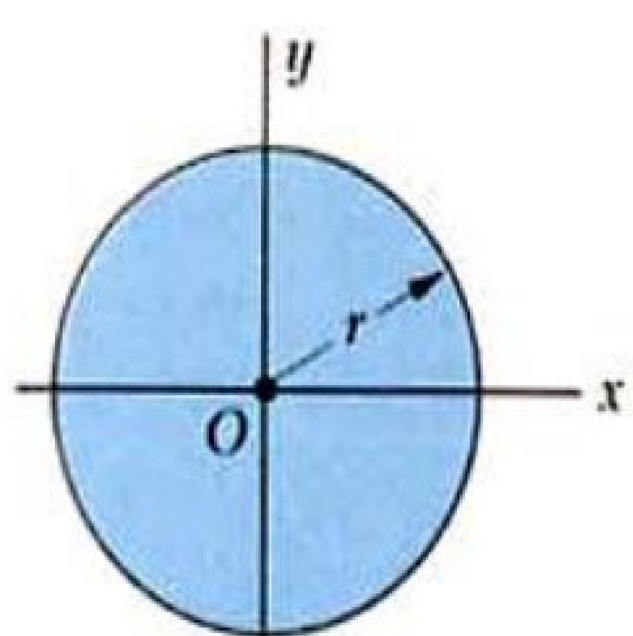
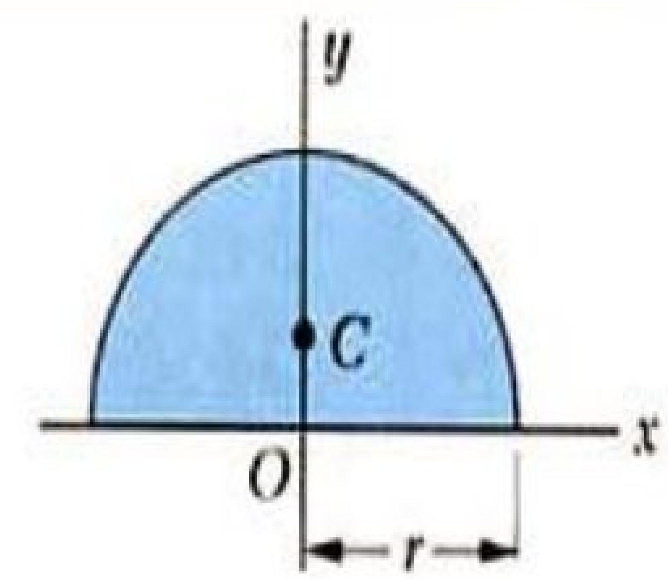
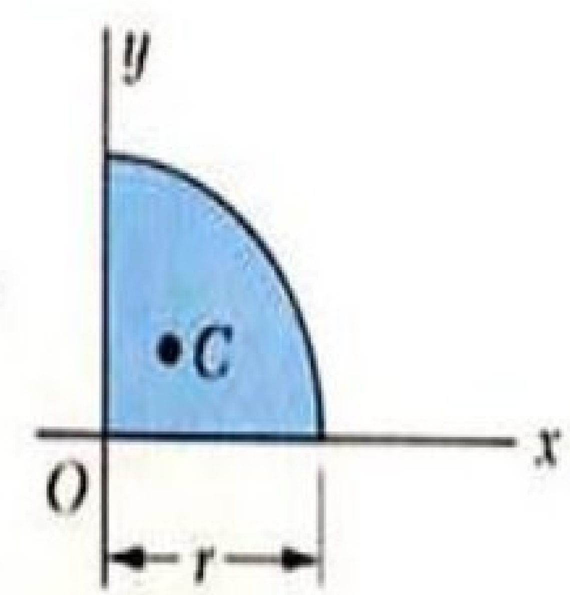
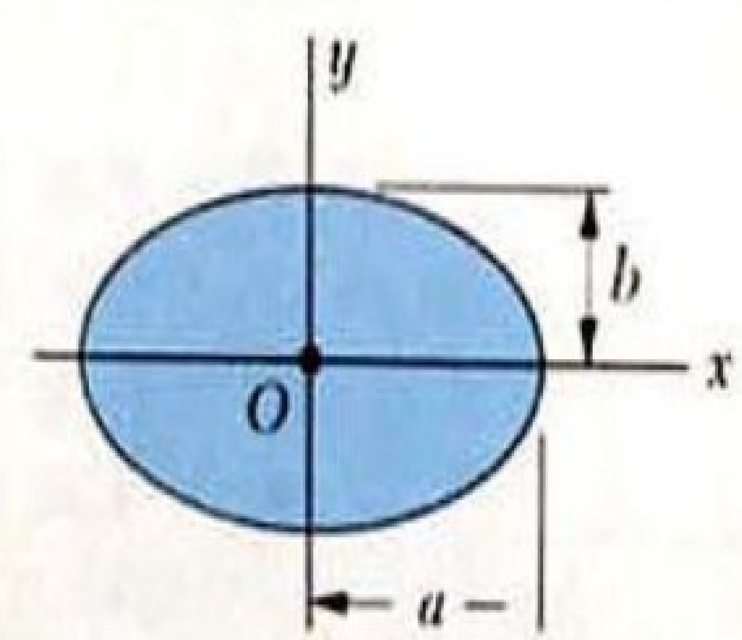
$$= \left[b \frac{y^3}{3} \right]_0^h = \frac{bh^3}{3}$$



Alternative Method

- Recall the method of finding centroids of composite bodies?
- Utilizing a know reference table we can use a similar tabulation technique to find the moment of inertia of any composite body.

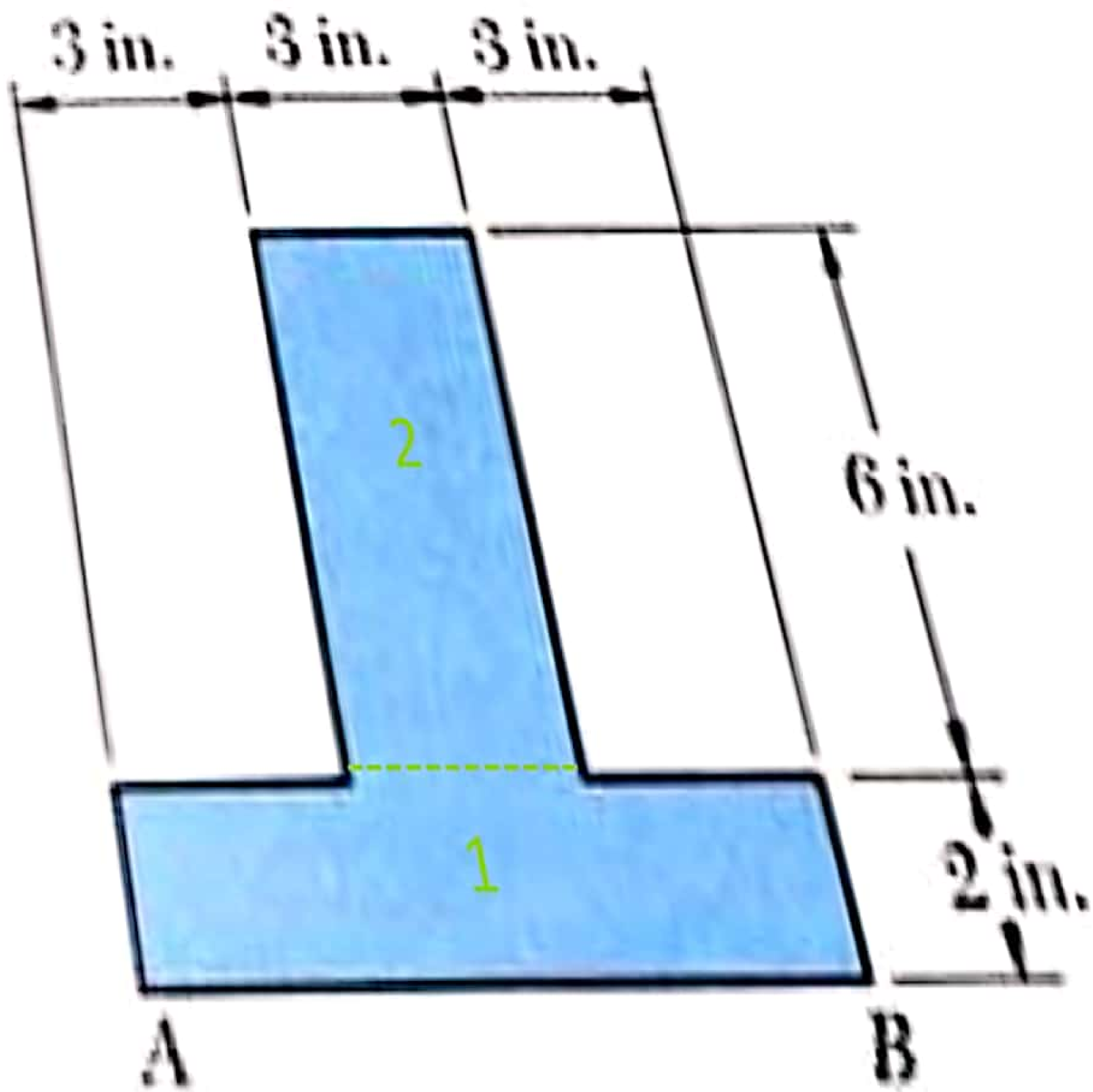
Standard Table Example

Rectangle		$\bar{I}_{x'} = \frac{1}{12}bh^3$ $\bar{I}_{y'} = \frac{1}{12}b^3h$ $I_x = \frac{1}{3}bh^3$ $I_y = \frac{1}{3}b^3h$ $J_C = \frac{1}{12}bh(b^2 + h^2)$
Triangle		$\bar{I}_{x'} = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$
Circle		$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$ $J_O = \frac{1}{2}\pi r^4$
Semicircle		$I_x = I_y = \frac{1}{8}\pi r^4$ $J_O = \frac{1}{4}\pi r^4$
Quarter circle		$I_x = I_y = \frac{1}{16}\pi r^4$ $J_O = \frac{1}{8}\pi r^4$
Ellipse		$\bar{I}_x = \frac{1}{4}\pi ab^3$ $\bar{I}_y = \frac{1}{4}\pi a^3b$ $J_O = \frac{1}{4}\pi ab(a^2 + b^2)$

Example

Find Moment of Inertia of this object:

- First we divide the object into two standard shapes present in the reference tables, the find the MI for each respective shape.

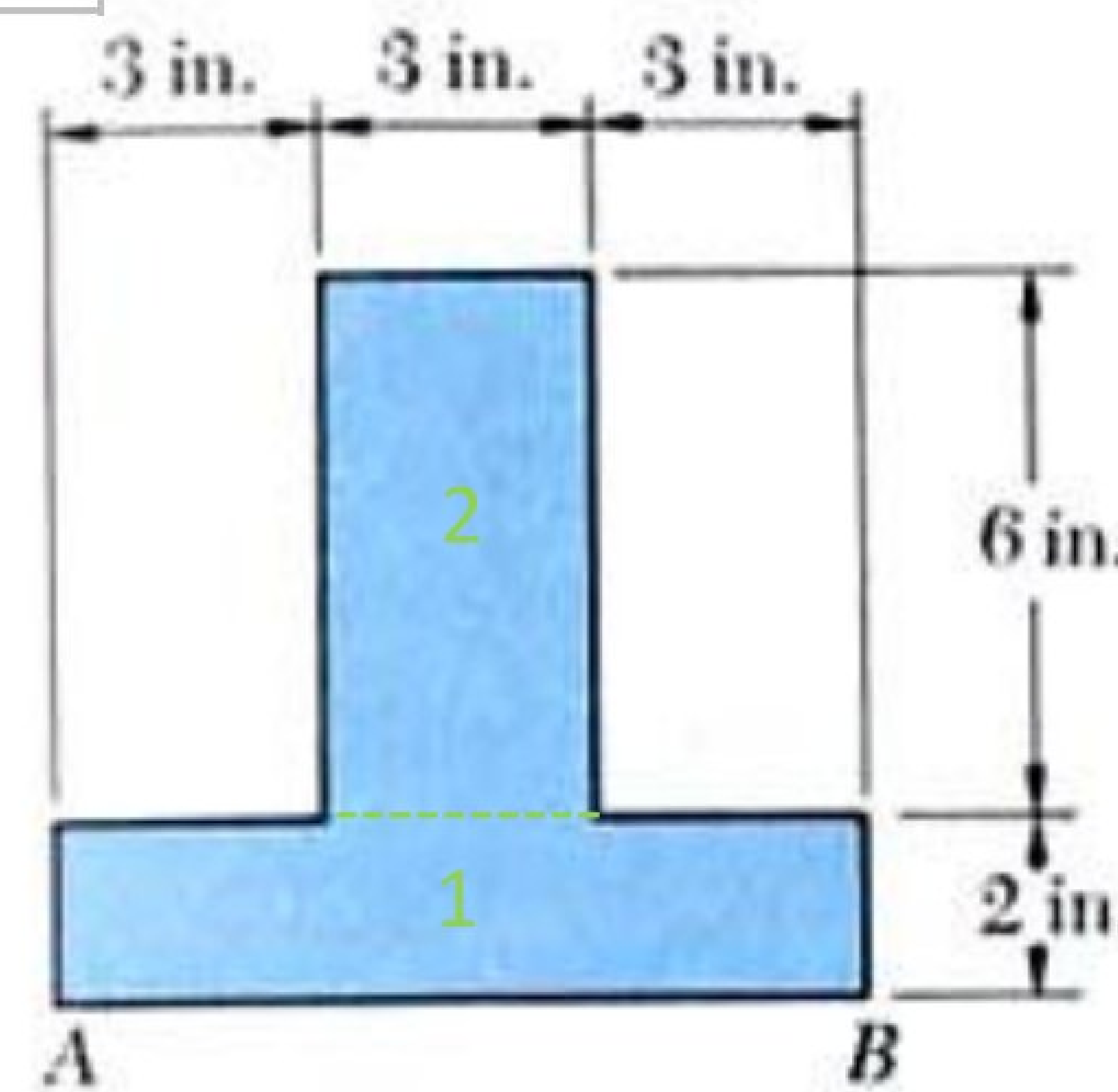


Example (cont'd)

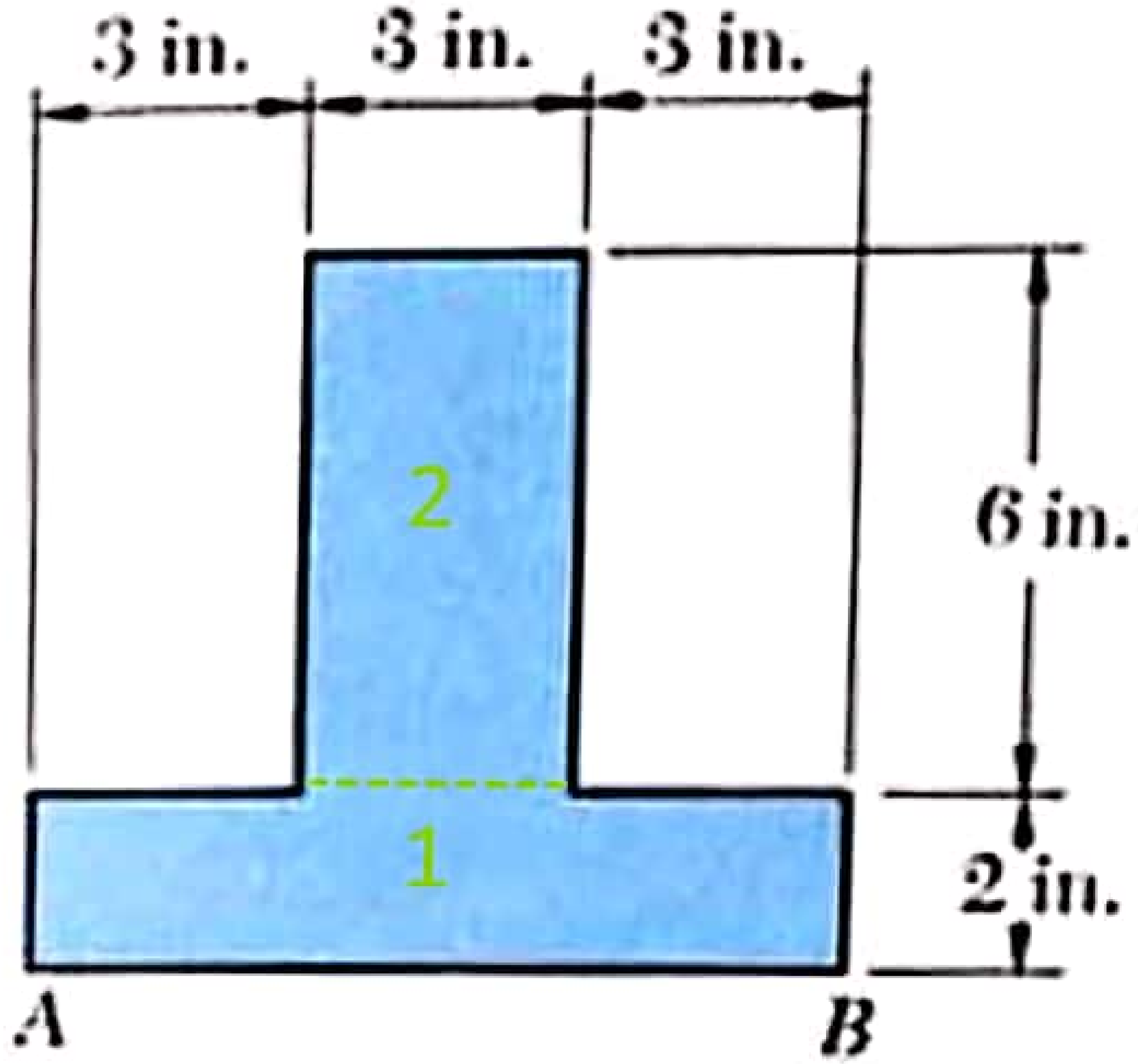
Set up the reference axis at AB and find the centroid:

Bodies	A_i	y_i	y_i*A_i	I_i	$d_i=y_i$-ybar	$d_i^2A_i$
1	18	1	18			
2	18	5	90			
	36		108			

$$\bar{y} = \frac{\sum y_i A_i}{\sum A_i} = \frac{108 \text{ in}^3}{36 \text{ in}^2} = 3.0 \text{ in.}$$



Example (cont'd)



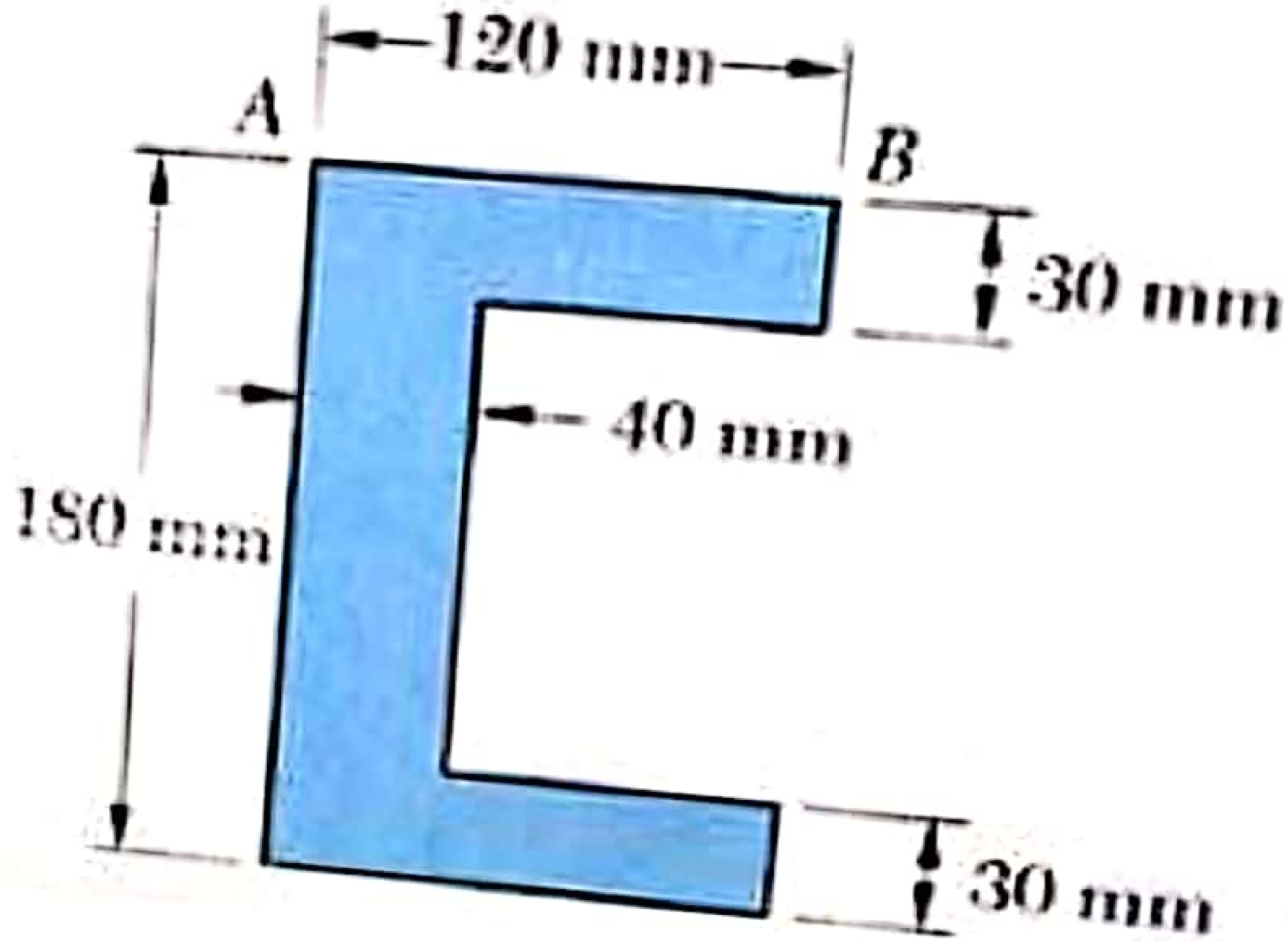
Find the moment of inertia from the table:

Bodies	A_i	y_i	$y_i{*}A_i$	I_i	$d_i=y_i$-ybar	$d_i^2A_i$
1	18	1	18	6	-2	72
2	18	5	90	54	2	72
	36		108	60		144
ybar	3	in.				
I	204	in^4				

$$I_x = \sum \overline{I_{xi}} + \sum \left(y_i - \bar{y}\right)^2 A_i$$

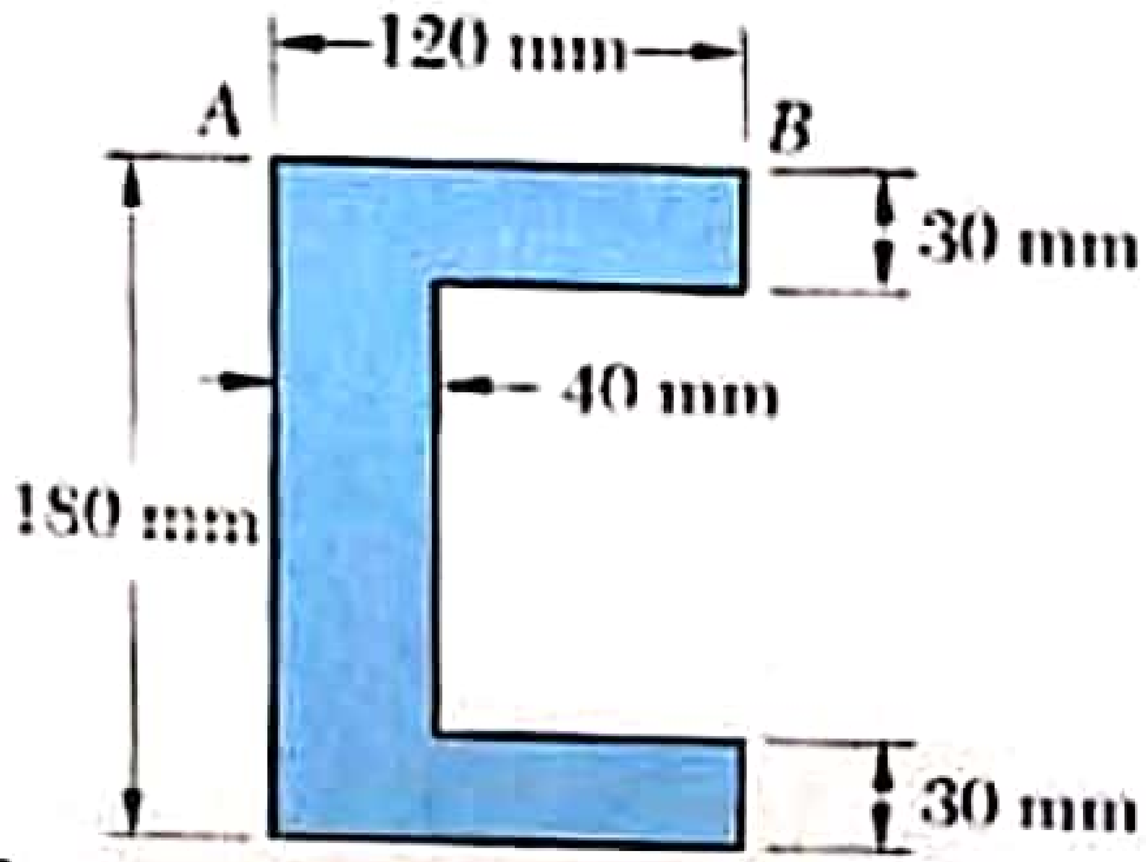
Example

Find the moment of inertia of the body using the same procedure illustrated previously.



Example (cont'd)

Find the MI of the whole rectangle (120mm*180mm) and then subtract the MI of the white rectangle (120mm*80mm) from the total area.



Bodies	A_i	y_i	y_i*A_i	I_i	$d_i=y_i$-ybar	$d_i^2A_i$
1	21600	90	1944000	58320000	0	0
2	-9600	90	-864000	-11520000	0	0
	12000		1080000	46800000		0
ybar	90	mm				
I	46800000	mm^4				

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